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PREDICTIVE OPTIMAL CONTROL FOR SEISMIC ANALYSIS OF NON-LINEAR AND HYSTERETIC STRUCTURES

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SUMMARY

In this paper, an effective active predictive control algorithm is developed for the vibration control of non-linear hysteretic structural systems subjected to earthquake excitation. The non-linear characteristics of the structural behaviour and the effects of time delay in both the measurements and control action are included throughout the entire analysis (design and validation). This is very important since, in current design practice, structures are assumed to behave non-linearly, and time delays induced by sensors and actuator devices are not avoidable. The proposed algorithm focuses on the instantaneous optimal control approach for the development of a control methodology where the non-linearities are brought into the analysis through a non-linear state vector and a non-linear open-loop term. An autoregressive (AR) model is used to predict the earthquake excitation to be considered in the prediction of the structural response. A performance index that is quadratic in the control force and in the predicted non-linear states, with two additional energy related terms, and that is subjected to a non-linear constraint equation, is minimized at every time step. The effectiveness of the proposed closed-open loop non-linear instantaneous optimal prediction control (CONIOPC) strategy is presented by the results of numerical simulations. Since non-linearity and time-delay effects are incorporated in the mathematical model throughout the derivation of the control methodology, good performance and stability of the controlled structural system are guaranteed. Copyright © 1999 John Wiley & Sons, Ltd.

KEY WORDS: non-linear optimal control; instantaneous control; predictive control; time delay; compensation; seismic response; non-linear structures; hysteretic structures; structural dynamics; performance index; ductility

1. INTRODUCTION

In recent years, intensive research efforts have been made to study the use of active control to suppress the vibrations of civil structures subjected to large environmental loads, such as impact, wind or earthquake loadings. Active control systems such as active tendons, active mass drivers, variable stiffness, etc., have been developed and tested in laboratory applications. In some cases, they have been implemented on full-scale building structures and have proven to be effective in improving the responses of such structures when subjected to environmental loads.[1]

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Various control methodologies have been investigated for civil structures such as buildings and bridges. For those systems, it is assumed that all operations in the control process can be performed instantaneously. However, in the real-time active control implementation, for a typical computer-controlled active system consisting of a set of interacting sensors and actuators, information about the state of the structural system (usually acceleration, velocity and displacement) first need to be measured and/or computed. Then, the computer defines the command signals according to the appropriate control methodology and finally the desired control force is delivered by means of an actuator. Therefore, time has to be accounted for data acquisition, data conditioning, on-line calculation of the control forces and their application. Thus, time delay causes unsynchronized application of the control forces which may not only degrade the performance of the control system but also even induce instability to the dynamic system.[2]

Research efforts on time delay effects in structural control of civil infrastructure systems have focused on linear structural models and control laws, such as linear quadratic optimal control, for the design of the controllers and actuators. Hence, all these control algorithms do not account for the non-linear hysteretic behaviour of structures when subjected to strong ground motion. However, recent large earthquakes have shown that the response of civil (steel and reinforced concrete) structures is highly non-linear, as a result of non-constant material properties, inelastic behaviour, large displacements, and progressive structural damage. It is evident that non-linearities play a key role in the response of structures subjected to seismic excitation and such effects should be included in the analysis and design of effective vibration control systems. As a result, at the First World Conference on Structural Control, held in Pasadena (CA) in 1994,[3] it was pointed out that non-linear controllers, considered the second generation of active structural control systems, should be studied for linear and non-linear civil engineering applications in order to capture the effects of the structural non-linearities in the design of appropriate control systems.

Various control methods have been investigated to study the behaviour of linear structures under seismic excitation. With the presence of time delay, the governing equations of an active structural control system are transferred from ordinary differential equations to differential–difference equations. Such time-delay control systems are hard to design and are not practically implementable. Soliman,[4] Mutharasan,[5] Hammerstrom,[6] and Abdel-Rohman[7] used Taylor series expansions to approximate the solution of such differential–difference equations. By using the assumption that structural vibrations are dominated by the natural frequencies of the structure, time-delay compensation is obtained in the phase plane by modifying the state feedback gain according to the phase lags of the displacement and velocity components of each vibration mode.[8,9] Pu and Kelly[10] and Inaudi and Kelly[11] addressed the use of phase compensation for active-isolation systems. This technique of phase compensation has been shown to be effective in both computer simulations and in laboratory experiments.[12–14] Abdel-Mooty[15] extrapolate the present system states from the delayed information in order to fit certain assumed time functions of the structural responses.

In this paper, following a non-linear instantaneous optimal control (NIOC) approach,[16] a predictive non-linear optimal controller is developed to reduce the peak response of seismically excited non-linear hysteretic structures. At each sampling time instant, the control action and the state response need to be predicted over a finite number of time steps ahead. Such a control force will be applied to the system with a known time delay and will produce a desired state response over the prediction interval. The earthquake excitation is predicted in a recursive form following an autoregressive representation.[17,18] A cost functional that is quadratic in the control force and in the predicted non-linear states and that is subjected to a non-linear constraint equation,

is defined and minimized at every time step. The resulting optimal controller is a predictive instantaneous controller in non-linear states whose gain matrices are determined by imposing the instantaneous optimality conditions on the predicted response. The advantage of such a controller is that the associated time-dependent optimal control gains are optimized at every time step accounting for the actual and predictive non-linear state of the structure. In this case, all sources of non-linearities, material as well as geometrical, will be included in the analysis.

2. EQUATION OF MOTION

The dynamic response of an N degree-of-freedom (NDOF) non-linear structural system, initially at rest and actively controlled by a closed–open loop controller, can be modelled by the following set of matrix equations of motion:

$$M\ddot{\mathbf{x}}(t) + \mathbf{f}_v(t) + \mathbf{f}_x(t) = D\mathbf{u}(t) + L\mathbf{f}(t) \tag{1}$$

in which $\mathbf{x}(t)$ is the n-dimensional displacement vector while $\mathbf{f}_v(t)$ and $\mathbf{f}_x(t)$ represent two n-dimensional internal force vectors related to the structural velocity and displacement, respectively. The vector $\mathbf{f}(t)$ indicates the r-dimensional external excitation vector: in the case of earthquake excitation, then $\mathbf{f}(t)$ reduces to $-MI\ddot{x}_g(t)$, where $\ddot{x}_g(t)$ is the earthquake-induced ground acceleration and I is the identity matrix. The matrix M represents the system's mass matrix, while D and L are the $n \times m$ control force location matrix and the $n \times r$ excitation location matrix, respectively. The vector $\mathbf{u}(t)$ represents the m-dimensional control force vector, which, in the most general case, can be expressed as

$$\mathbf{u}(t) = G_x(t)\mathbf{x}(t) + G_{\dot{x}}(t)\dot{\mathbf{x}}(t) + G_f(t)\mathbf{f}(t) \tag{2}$$

where $G_x(t)$, $G_{\dot{x}}(t)$ and $G_f(t)$ are the unknown control gains. Equation (2) shows that the control terms related to the non-linear response $\mathbf{x}(t)$ and $\dot{\mathbf{x}}(t)$ are fed back, through the controller, to the structure and form the closed-loop control component. The external excitation is also fed forward to the system to form the open-loop control. Previous studies[19] have shown the effectiveness of including an open-loop component in the control action. Substituting the control law expressed by equation (2) into the equations of motion (equation (1)) leads to the 'controlled' equations of motion:

$$M\ddot{\mathbf{x}}(t) + [\mathbf{f}_v(t) - DG_{\dot{x}}(t)\dot{\mathbf{x}}(t)] + [\mathbf{f}_x(t) - DG_x(t)\mathbf{x}(t)] = [DG_f(t) + L]\mathbf{f}(t) \tag{3}$$

The goal of this study is to find the 'optimal' values of the control gain matrices $G_x(t)$, $G_{\dot{x}}(t)$ and $G_f(t)$ which will minimize an approximately defined cost functional and will satisfy equation (3). The effects of the non-linearities of the structure are explicitly included in the optimization process through the components $\mathbf{f}_x(t)$ and $\mathbf{f}_v(t)$. At a given time t, the proposed instantaneous optimization process will account for the non-linear structural response up to that instant, and so will the consequent corrective control action.

Using a state-space representation, equation (1) can be written as

$$\dot{\mathbf{z}}(t) = \mathbf{f}_z(\mathbf{z}) + B\mathbf{u}(t) + H\ddot{x}_g(t) \tag{4}$$

with initial conditions

$$\mathbf{z}(0) = 0 \tag{5}$$

where the state vector $\mathbf{z}(t)$, the system vector $\mathbf{f}_z(\mathbf{z})$, the controller location matrix B and the external excitation location matrix H are expressed as

$$\mathbf{z}(t) = \begin{bmatrix} \mathbf{x}(t) \\ \dot{\mathbf{x}}(t) \end{bmatrix} \tag{6}$$

$$\mathbf{f}_z(\mathbf{z}) = \begin{bmatrix} \dot{\mathbf{x}}(t) \\ -M^{-1}\mathbf{f}_v(t) - M^{-1}\mathbf{f}_x(t) \end{bmatrix} \tag{7}$$

$$B = \begin{bmatrix} 0 \\ -M^{-1}D \end{bmatrix} \tag{8}$$

$$H = \begin{bmatrix} 0 \\ -M^{-1}L \end{bmatrix} \tag{9}$$

In the continuous time formulation, the state equation (equation (4)) represents a set of differential equations. However, since the earthquake ground acceleration data are in discrete form and the control action is implemented using a digital controller, it is necessary to formalize the system's equations of motion in the discrete time domain to carry out the numerical simulations. In a discrete time formulation, continuous time functions are replaced by discrete time functions, which can be evaluated at any discrete time interval multiple of a chosen time step Δt. The equations of motion are then transformed from differential equations with state-space variables $\mathbf{z}(t)$ to difference equations in terms of the discrete state vector $\mathbf{z}_k$ at the kth time step. The external excitation $\mathbf{f}(t)$ is also expressed as $-MI\ddot{x}_{gk}$, where $\ddot{x}_{gk}$ represents the ground acceleration at time $k\Delta t$.

To discretize the continuous time state space into the discrete time state space properly, the sampling rate and the frequency content of the external earthquake need to be analysed. Since the sampling rate of the earthquake excitation records is 0·01 s (corresponding to a cutoff frequency of 100 Hz), and civil structures have natural frequencies far below 100 Hz, the zero-order-hold sampling assumption is used. The error introduced by the discretization may be made negligible by using a sampling time step sufficiently small compared with the significant time constant of the system. In this case, the two representations are then equivalent.

In order to provide a more realistic structural representation of the control action, the time-delay effects have to be included into the analysis. Time delays are associated with (1) the process of digitizing the observed input motion and system response, and (2) the calculation of the control force and its application in the form of stepwise functions through A/D converters. In this study, the two types of time delays have been considered, both jointly and separately, so that their individual effects could be highlighted. In this analysis, the two types of time delays are indicated with $\lambda\Delta t$ and $\delta\Delta t$, respectively. Introducing the time-delay effects will imply that, at time $k\Delta t$, the available data are those measured at time $(k-\lambda)\Delta t$ and that the control force determined at time $k\Delta t$ will be applied to the structure at time $(k+\delta)\Delta t$. This will require the prediction of the structural response and of the ground excitation over a time interval $(\lambda+\delta)\Delta t$ and will impose

a differentiation between actual structural response and predicted structural response, which will be indicated with an overline.

By using Runge–Kutta fourth order approximation,[20] the discrete time state-space representation of the equations of motion (equation (4)), including the effects of time-delay compensation, can be developed for both the actual and the predicted response. In terms of the actual structural response, the state vector at time $(k+1)\Delta t$ can be expressed as

$$\mathbf{z}_{k+1} = \mathbf{D}_k + \frac{\Delta t}{6}[B\mathbf{u}_{k-\delta+1} + H\ddot{x}_{gk+1}] \tag{10}$$

where $\mathbf{D}_k$ is a vector defined at time $k\Delta t$ and containing all known quantities:

$$\mathbf{D}_k = \mathbf{z}_k + \frac{\Delta t}{6}[\mathbf{\Xi}_k + 5B\mathbf{u}_{k-\delta} + 5H\ddot{x}_{gk}] \tag{11}$$

$$\mathbf{\Xi}_k = \mathbf{f}_z(\mathbf{z}_k) + 2\mathbf{f}_z(\mathbf{z}_k + \tfrac{1}{2}\mathbf{K}_1^r) + 2\mathbf{f}_z(\mathbf{z}_k + \tfrac{1}{2}\mathbf{K}_2^r) + \mathbf{f}_z(\mathbf{z}_k + \mathbf{K}_3^r) \tag{12}$$

with

$$\mathbf{K}_1^r = \Delta t\, \mathbf{S}(t_k, \mathbf{z}_k, \mathbf{u}_{k-\delta}) \tag{13}$$

$$\mathbf{K}_2^r = \Delta t\, \mathbf{S}(t_{k+1/2}, \mathbf{z}_k + \tfrac{1}{2}\mathbf{K}_1^r, \mathbf{u}_{k-\delta}) \tag{14}$$

$$\mathbf{K}_3^r = \Delta t\, \mathbf{S}(t_{k+1/2}, \mathbf{z}_k + \tfrac{1}{2}\mathbf{K}_2^r, \mathbf{u}_{k-\delta}) \tag{15}$$

and

$$\mathbf{S}(t_k, \mathbf{z}_k) = \mathbf{f}_z(\mathbf{z}_k) + B\mathbf{u}_{k-\delta} + H\ddot{x}_{gk} \tag{16}$$

The term in equation (10) associated with the control force, $\mathbf{u}_{k-\delta+1}$, indicates the control action that, determined at time $(k-\delta+1)\Delta t$, will affect the structural response at time $(k+1)\Delta t$. The optimal gains of such a control action will be obtained using the instantaneous optimal control approach proposed in this study.

However, when the time-delay effects in the measurements and control action are considered, predictions of both the structural response and the ground excitation from $(k-\lambda+1)\Delta t$ to $(k+\delta)\Delta t$ must be provided. This will require another set of equations of motion in terms of the predicted structural response, $\bar{\mathbf{z}}$, and earthquake excitation, $\bar{\ddot{x}}$, which can be expressed as

$$\bar{\mathbf{z}}_{i+1} = \bar{\mathbf{D}}_i + \frac{\Delta t}{6}[B\mathbf{u}_{i-\delta+1} + H\bar{\ddot{x}}_{gi+1}] \tag{17}$$

with $\bar{\mathbf{D}}_i$ defined as

$$\bar{\mathbf{D}}_i = \bar{\mathbf{z}}_i + \frac{\Delta t}{6}[\bar{\mathbf{\Xi}}_i + 5B\mathbf{u}_{i-\delta} + 5H\bar{\ddot{x}}_{gi}] \tag{18}$$

where the index i goes from $(k-\lambda+1)$ to $(k+\delta)$. The expressions for the overlined quantities in equations (17) and (18) are identical to those presented in equations (12)–(16), with the exception that the structural response and the ground excitation correspond to the predicted ones ($\bar{\mathbf{z}}$, $\bar{\ddot{x}}$).

It is worthwhile to note that, while equation (10) is just a one-step ahead process, equation (17) requires $(\lambda + \delta)$-step prediction iterations.

3. OPTIMIZATION OF THE DYNAMIC NON-LINEAR STRUCTURAL RESPONSE

In forming an optimal control problem, it is necessary to create a cost functional which includes, appropriately weighted, state and control variables, and system parameters. When dealing with non-linear hysteretic structures, in addition to all the conventional requirements, the non-linear properties of the structural system must also be considered. The optimal control problem should then consist in finding a control sequence $\mathbf{u}_k$ that satisfies the appropriate equations of motion and minimizes the prescribed performance cost functional. Therefore, minimizing such a performance index with respect to the control and state variables and constrained by the equations of motion, defines an optimal solution.

Since non-linear structures present a behaviour which is strongly path-dependent, it is then appropriate to define a time-dependent (instantaneous) quadratic performance cost functional J_k at time $k\Delta t$ that must be minimized at the kth time step. A procedure of this type is called 'Instantaneous Optimal Control' and a commonly used cost functional is represented by

$$J_{k+1} = \mathbf{z}_{k+1}^{\mathrm{T}} Q \mathbf{z}_{k+1} + \mathbf{u}_{k+1}^{\mathrm{T}} R \mathbf{u}_{k+1} \tag{19}$$

The matrix Q represents a positive semi-definite weighting matrix related to the structural state, while R is a positive definite weighting matrix for the control force. However, a control methodology based on this type of functionals has the characteristic that open, closed and closed–open loop control systems show identical efficiency.[21] From previous studies on classical optimal control systems based on ensemble training,[19] it appears that the closed–open loop optimal approach provides better performances in terms of reduction of maximum structural response and peak control force. It is then desirable to apply the same closed–open loop control methodology also for an instantaneous optimal control approach when applied to the case of non-linear structures.

If time delays are included in the analysis without proper compensation, the control force will induce instability in the structural response. In fact, the control force is obtained from the optimization process at a specific time $k\Delta t$ but it will be applied to a different time step at which it is not the optimal control force anymore. As a consequence, the structure will be subject to an additional external force which could cause instability. It is then necessary to properly compensate for the time-delay effects.

In order to derive an instantaneous optimal control approach that is based on an effective closed–open loop scheme and that accounts for time-delay compensation for both the measurements and the control action, we introduce a new quadratic cost functional $J^p_{k+\delta+1}$ defined at time $k\Delta t$, which must be minimized over the time interval $k\Delta t - (k+1)\Delta t$. The reason of the index $(k+\delta+1)$ is that, at time $k\Delta t$, we are optimizing the control action to the next time step which, because of the time delay in the control action, will affect the structural response at time $(k+\delta+1)\Delta t$. At time $k\Delta t$, the new quadratic cost functional $J^p_{k+\delta+1}$ can be expressed as

$$J^p_{k+\delta+1} = \mathbf{Y}^{\mathrm{T}}_{k+\delta+1} \Theta \mathbf{Y}_{k+\delta+1} \tag{20}$$

in which the vector $\mathbf{Y}_{k+\delta+1}$ and matrix Θ are defined as

$$\mathbf{Y}_{k+\delta+1} = \begin{bmatrix} \bar{\mathbf{z}}_{k+\delta+1} \\ \mathbf{u}_{k+1} \\ \ddot{\bar{x}}_{gk+\delta+1} \end{bmatrix} \tag{21}$$

$$\Theta = \begin{bmatrix} Q & \frac{1}{2}F & \frac{1}{2}F \\ \frac{1}{2}F^{\mathrm{T}} & R & \frac{1}{2}V \\ \frac{1}{2}E^{\mathrm{T}} & \frac{1}{2}V^{\mathrm{T}} & W \end{bmatrix} \tag{22}$$

and the matrices Q, R, E, F, W and V are considered as weighting matrices.

The weighting matrices Q and R are identical to those used in classical optimal control, with the only requirement that Q must be positive semi-definite and R positive definite. In this way, the quadratic terms associated with these two matrices represent the 'classical' cost functional J_{k+1} defined in equation (19), including time delay compensation, and are referred to the instantaneous values of the system's strain energy and kinetic energy. The two matrices Q and R have been chosen as

$$Q = \zeta \frac{Q^*}{\Delta t}, \qquad R = \eta R^* \Delta t$$

in order to avoid the problem that the control force tends to zero when the Δt becomes very small. The magnitude of the elements of Q^* and R^* and of the parameters ζ and η is assigned according to the relative importance attached to the state variables and to the control forces in the minimization procedure. The weighting matrix W relates to a quadratic form of the ground acceleration which is independent of the control system and so it should not be included in the optimization process. For this reason, the matrix W is assumed to be a null matrix. The matrices E and F are related to the cross terms between the predicted state and ground acceleration and the predicted state and the applied control force, respectively. These cross products can also be viewed as energy terms accounting for the incremental work done on the structural system by the earthquake action ($\dot{\bar{x}}^{\mathrm{T}}_{k+\delta+1} \Delta t M I \ddot{\bar{x}}_{k+\delta+1}$) and by the control force ($\dot{\bar{x}}^{\mathrm{T}}_{k+\delta+1} \Delta t \mathbf{u}_{k+1}$) during the time interval $(k+\delta+1)\Delta t$. In this way, the newly defined matrices E and F will then be chosen as

$$E = \begin{bmatrix} 0 \\ -MI\Delta t \end{bmatrix} \tag{23}$$

$$F = \begin{bmatrix} 0 \\ \Delta t \end{bmatrix} \tag{24}$$

Since the weighting matrix V couples the control force at time $(k+1)\Delta t$, $\mathbf{u}_{k+1}$, and the predicted ground acceleration at time $(k+\delta+1)\Delta t$, $\ddot{\bar{x}}_{gk+\delta+1}$, this cross product does not have any energy related significance and can be dropped out from the cost functional. Hence, the weighting matrix V can be assumed to be a null matrix.

After mathematical manipulations, the cost functional $J^p_{k+\delta+1}$ in equation (20) can be expressed as:

$$J^p_{k+\delta+1} = \bar{\mathbf{z}}^{\mathrm{T}}_{k+\delta+1} Q \bar{\mathbf{z}}_{k+\delta+1} + \mathbf{u}^{\mathrm{T}}_{k+1} R \mathbf{u}_{k+1} + \bar{\mathbf{z}}^{\mathrm{T}}_{k\delta+1} E \bar{\ddot{x}}_{\mathrm{g}k+\delta+1} + \bar{\mathbf{z}}^{\mathrm{T}}_{k+\delta+1} F \mathbf{u}_{k+1} \tag{25}$$

which has to be minimized at every time instant $(k+\delta+1)\Delta t$ for all $\lambda \Delta t \leqslant (k+\delta+1)\Delta t \leqslant t_f + \delta \Delta t^1$ with t_f being the time duration of the seismic analysis. The selection of such matrices allows us to (1) prove that the proposed cost functional is positive semidefinite, and (2) guarantee the stability of the controlled non-linear structural system, when using a Lyapunov direct approach.[22]

In order to minimize the performance index $J^p_{k+\delta+1}$ given by equation (25) subjected to the constraint expressed by equation (17), the Hamiltonian $\psi_{k+\delta+1}$ at time $(k+\delta+1)\Delta t$ is obtained as follows:

$$\Psi_{k+\delta+1} = J^p_{k+\delta+1} + \lambda^{\mathrm{T}}_{k+\delta+1} \left\{ \bar{\mathbf{z}}_{k+\delta+1} - \bar{\mathbf{D}}_{k+\delta} - \frac{\Delta t}{6} [B \mathbf{u}_{k+1} + H \bar{\ddot{x}}_{\mathrm{g}k+\delta+1}] \right\} \tag{26}$$

where $\lambda_{k+\delta+1}$ is the costate vector at time instant $(k+\delta+1)\Delta t$. In a discrete time formulation, such a defined Hamiltonian can be considered as a function of discrete independent variables (assumed uncorrelated), which reduces the optimality conditions at time $(k+\delta+1)\Delta t$ in the form:

$$\frac{\partial \Psi_{k+\delta+1}}{\partial \bar{\mathbf{z}}_{k+\delta+1}} = \mathbf{0} \tag{27}$$

$$\frac{\partial \Psi_{k+\delta+1}}{\partial \mathbf{u}_{k+1}} = \mathbf{0} \tag{28}$$

$$\frac{\partial \Psi_{k+\delta+1}}{\partial \lambda_{k+\delta+1}} = \mathbf{0} \tag{29}$$

Imposing these 'instantaneous optimality conditions', the control vector $\mathbf{u}_{k+1}$ and the predicted response state vector $\bar{\mathbf{z}}_{k+\delta+1}$ can be determined as

$$P = \left[I + \frac{\Delta t}{6} B \left(2R + \frac{\Delta t}{6} B^{\mathrm{T}} F \right)^{-1} \left(F^{\mathrm{T}} + \frac{\Delta t}{3} B^{\mathrm{T}} Q \right) \right]^{-1} \tag{30}$$

$$\bar{\mathbf{z}}_{k+\delta+1} = P \left[\bar{\mathbf{D}}_{k+\delta} + \left[\frac{\Delta t}{6} H - \left(\frac{\Delta t}{6} \right)^2 B \left(2R + \frac{\Delta t}{6} B^{\mathrm{T}} F \right)^{-1} B^{\mathrm{T}} E \right] \right] \bar{\ddot{x}}_{\mathrm{g}k+\delta+1} \tag{31}$$

$$\mathbf{u}_{k+1} = \left(2R + \frac{\Delta t}{6} B^{\mathrm{T}} F \right)^{-1} \left[- \left(F^{\mathrm{T}} + \frac{\Delta t}{3} B^{\mathrm{T}} Q \right) \bar{\mathbf{z}}_{k+\delta+1} - \frac{\Delta t}{6} B^{\mathrm{T}} E \bar{\ddot{x}}_{\mathrm{g}k+\delta+1} \right] \tag{32}$$

In these equations, the control force $\mathbf{u}_{k+1}$ is a function of the predicted values of the state, $\bar{\mathbf{z}}_{k+\delta+1}$, and of the ground acceleration, $\bar{\ddot{x}}_{\mathrm{g}k+\delta+1}$, at time $(k+\delta+1)\Delta t$. This prediction of earthquake excitation is carried out using an autoregressive (AR) representation as presented in the work of Sato and Toki.[21] At time $(k+\delta+1)\Delta t$, the predicted ground acceleration can be

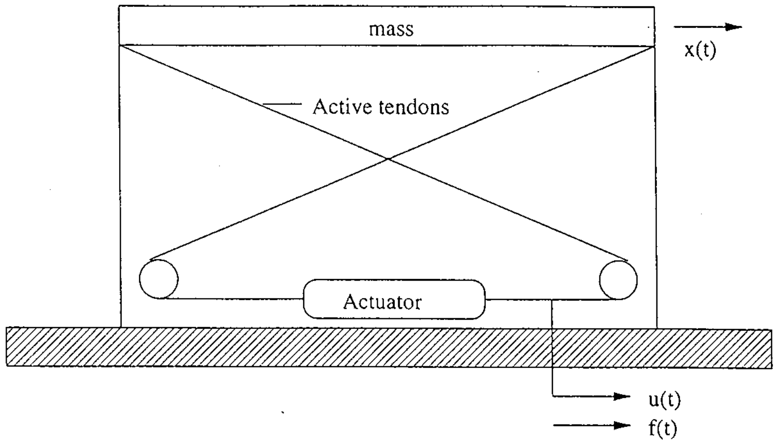


Figure 1. SDOF structural model

expressed as:

$$\bar{\ddot{x}}_{gk+\delta+1} = -\phi_1 \bar{\ddot{x}}_{gk+\delta} - \phi_2 \bar{\ddot{x}}_{gk+\delta-1} \cdots - \phi_{\delta+\lambda+1} \ddot{x}_{gk-\lambda} - \phi_{\delta+\lambda+2} \ddot{x}_{gk-\lambda-1} \cdots + v_m \tag{33}$$

where v_m is white noise, and ϕ_i are parameters which are identified recursively by using the Kalman filter method. It should be noted that, up to time $k\Delta t$, the values of ground acceleration up to time $(k-\lambda)\Delta t$ are from real measurements while the other values are obtained from this recursive prediction algorithm. First, the state space of the parameters ϕ of the autoregressive process needs to be defined. By using the observation equation (equation (33)), a Kalman filter equation is formulated and its gains are obtained by estimating the error covariance of the state. In this way, the earthquake excitation and, consequently, the structural response at future times can be estimated.

Once the control force has been obtained, it will then be used in equation (10) to determine the actual response of the structural system at time $(k+\delta+1)\Delta t$.

4. ANALYSIS OF THE NUMERICAL RESULTS

4.1. *Definition of structural models and ground input motion*

To investigate the effects of time-delay compensation on the effectiveness of the proposed instantaneous optimal control approach for the seismic analysis of non-linear structures, numerical simulations have been conducted using a single degree-of-freedom (SDOF) structural model.

This model is a SDOF linear model with an active tendon control system, adopted from Soong[1] and shown in Figure 1. This model was chosen because it has been extensively used in previous studies as a reference frame for the comparison of the results. Its structural properties are summarized as follows: the structural mass $m = 2923$ kg m, the structural stiffness $k = 1389$ kN/m, the tendon stiffness $k_c = 372$ kN/m, the tendon angle $\gamma = 36°$. The natural frequency of the uncontrolled structure is 3·47 Hz.

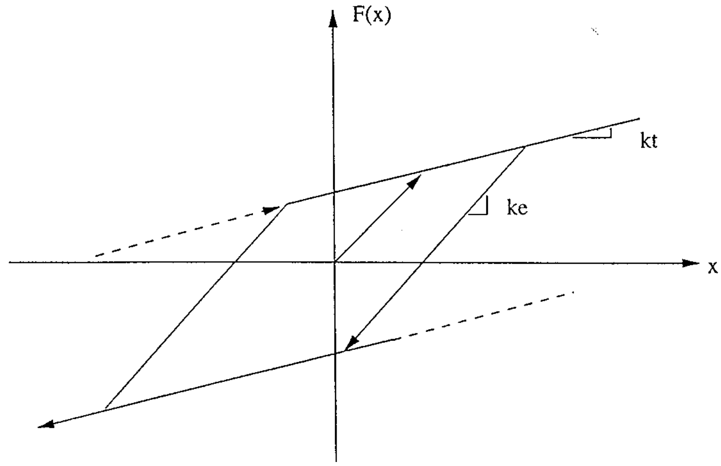


Figure 2. Non-linear force–deformation relationship

Table I. Maximum response quantities for model 1: equal weighting matrices

	Uncontrolled		IOC		CONIOC	
	Linear	Non-linear	Linear	Non-linear	Linear	Non-linear
x (m)	0·0278	0·0236	0·00171	0·00172	0·00068	0·00068
$\ddot{x}$ (g)	1·358	0·512	0·078	0·078	0·0295	0·0295
ρ	N/A (18·54)	15·73	N/A (1·14)	1·148	N/A (0·453)	0·453
F_c (N)	0·0	0·0	7439	7318	7438	7438

However, in order to make the structure exhibit a non-linear behaviour when subjected to strong earthquake loadings, the properties of such a structural model have been modified. The structural stiffness has been assumed to be bilinear elastoplastic with an elastic stiffness $k_e =$ 1389 kN/m and a hardening post elastic tangent stiffness $k_t = \frac{1}{3} k_e$, as shown in Figure 2. Yielding is assumed to occur for a lateral displacement of 0·0015 m. Such a value has been chosen so that even actively controlled structures will present a non-linear behaviour, allowing us to test the effectiveness of active control systems in the non-linear range.

For this structural model, the components of the internal forces associated with the structural velocity, $\mathbf{f}_v$, has been assumed equal to $(2\xi\sqrt{k_e m})\,\dot{x}$, where ξ represents the damping ratio. For this model, the value of ξ has been chosen equal to 0·0124, as reported in Reference 1.

In order to investigate the structural response to strong earthquake excitation, this structural model has been subjected to the recorded ground motion of the 1994 Northridge earthquake. The ground motion generated by this event has been responsible for extensive structural damage in moment-resisting structures, providing then a good test case for the non-linear structural behaviour.

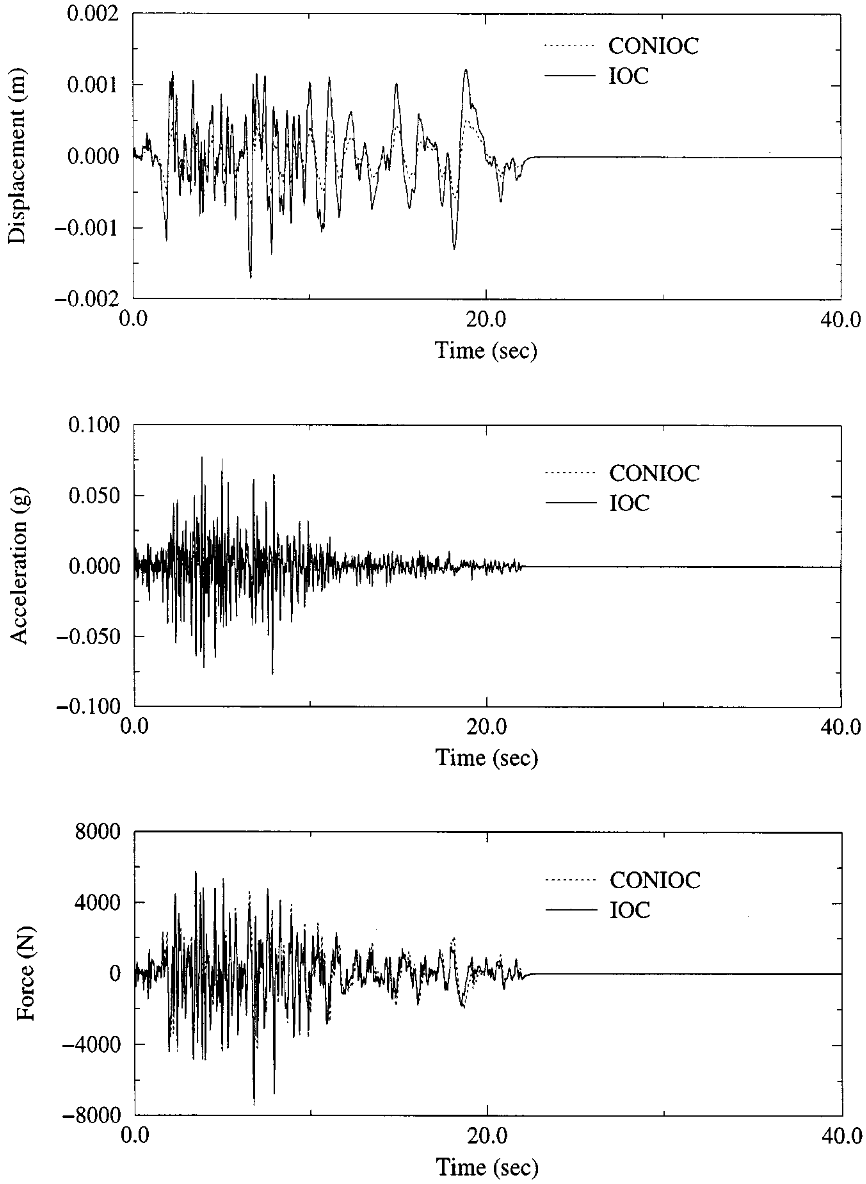


Figure 3. Structural responses and control force for linear structure

4.2. *Active control methodologies*

In this study, two different active control methodologies have been used for the analysis of the response of linear and bilinear structural models to earthquake excitation. The first control system considered in this study is based on the proposed active instantaneous optimal control

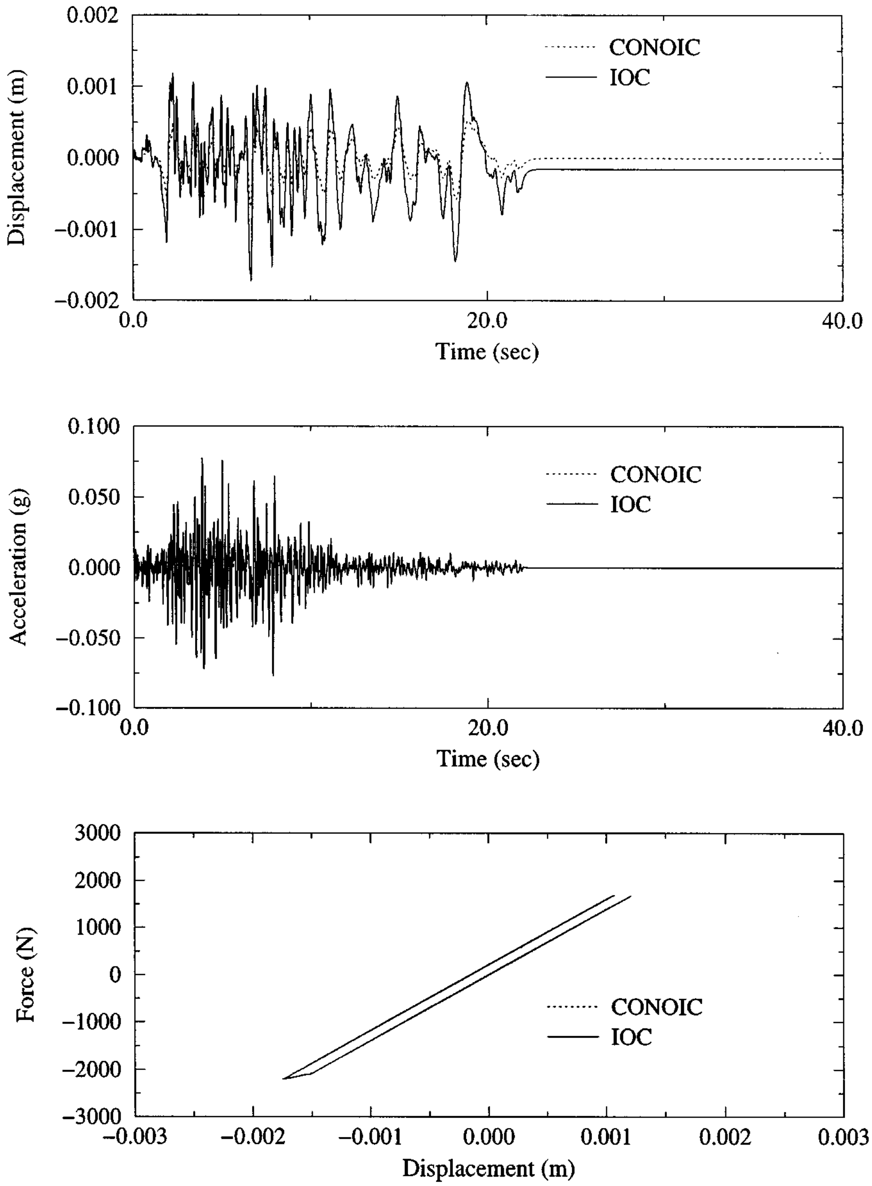


Figure 4. Structural responses and hysteresis loop for bilinear structure

approach which automatically includes the non-linearities of the structure and accounts for time delay compensation (i.e. close–open non-linear instantaneous optimal prediction control: CON-IOPC). The second system is represented by the classic instantaneous optimal control (IOC) algorithm proposed by Yang.[23] Since such a method is well established and effective in the vibration control of structural systems, it is used here as the reference frame.

Table II. Maximum response, ductility, control force under different λ and δ for bilinear model 1

$\lambda + \delta + 1$	1	2	3	4	5	6	7	8
x (m)	0·00295	0·00416	0·00633	0·00819	0·00985	0·01140	0·01280	0·01390
$\ddot{x}$ (g)	0·124	0·161	0·213	0·257	0·292	0·322	0·327	0·328
ρ	1·968	2·773	4·222	5·458	6·565	7·618	8·518	9·230
F_c (N)	7107	6968	6877	7291	7616	7754	7666	7433
Sp_x (m)	0·0086	0·0110	0·0158	0·0210	0·0260	0·0301	0·0334	0·0359
$Sp_{\ddot{x}}$ (g)	0·539	0·689	0·948	1·149	1·300	1·430	1·560	1·620

Table III. Maximum response, ductility, control force under different λ and δ for bilinear model 1

$\lambda + \delta + 1$	1	2	3	4	5	6	7	8
x (m)	0·00296	0·00379	0·00535	0·00668	0·00775	0·00849	0·00888	0·00892
$\ddot{x}$ (g)	0·123	0·161	0·212	0·249	0·268	0·273	0·273	0·273
ρ (N/A)	1·969	2·526	3·565	4·457	5·164	5·658	5·917	5·947
F_c (N)	8151	8489	8926	9418	9883	10451	10866	10920
Sp_x (m)	0·0089	0·0115	0·0162	0·0204	0·0237	0·0261	0·0273	0·0275
$Sp_{\ddot{x}}$ (g)	0·549	0·694	0·960	1·164	1·310	1·380	1·396	1·395

4.3. *Control action with equal weighting matrices*

Let us now consider the effectiveness of the proposed control algorithm in terms of reduction of peak structural responses. First, we assume that the weighting matrices Q and R have the same numerical values for both control methodologies so that it is possible to compare their performances with identical trade-off between structural response and control force.

In the case of linear behaviour and no time delays, both control algorithms are quite effective. From Table I and Figure 3, the peak structural displacement response is drastically reduced from 0·0278 m (uncontrolled) to 0·00171 m (IOC) and 0·00068 m (CONIOPC), and so is the maximum floor acceleration (1·358g (uncontrolled); 0·078g (IOC); 0·030g (CONIOPC)). On the other hand, it is noteworthy that, although the peak displacement and acceleration responses decrease from IOC to CONIOPC method (51 per cent for the displacements and 164 per cent for the accelerations), the maximum values of the control force are almost identical (Table I). The reasons for an improved performance by the CONIOPC approach can be found in two factors: (1) the control gains are obtained from the optimization of a different cost functional which includes additional energy terms, and (2) the proposed control algorithm has an open-loop component which contributes to the reduction of the external excitation (see equation (3)).

A similar reduction pattern can be seen for the cases when the structural models are allowed to exhibit a non-linear behaviour (Table I) and Figure 4. It can be seen that the CONIOPC approach is effective in keeping the structure within the elastic range (no residual deformations) while the classical IOC allows minimum plastic deformations ($\rho = 1{\cdot}148$). As expected, since the CONIOPC approach keeps the structure elastic, the required maximum control force is larger than the one required by the IOC approach.

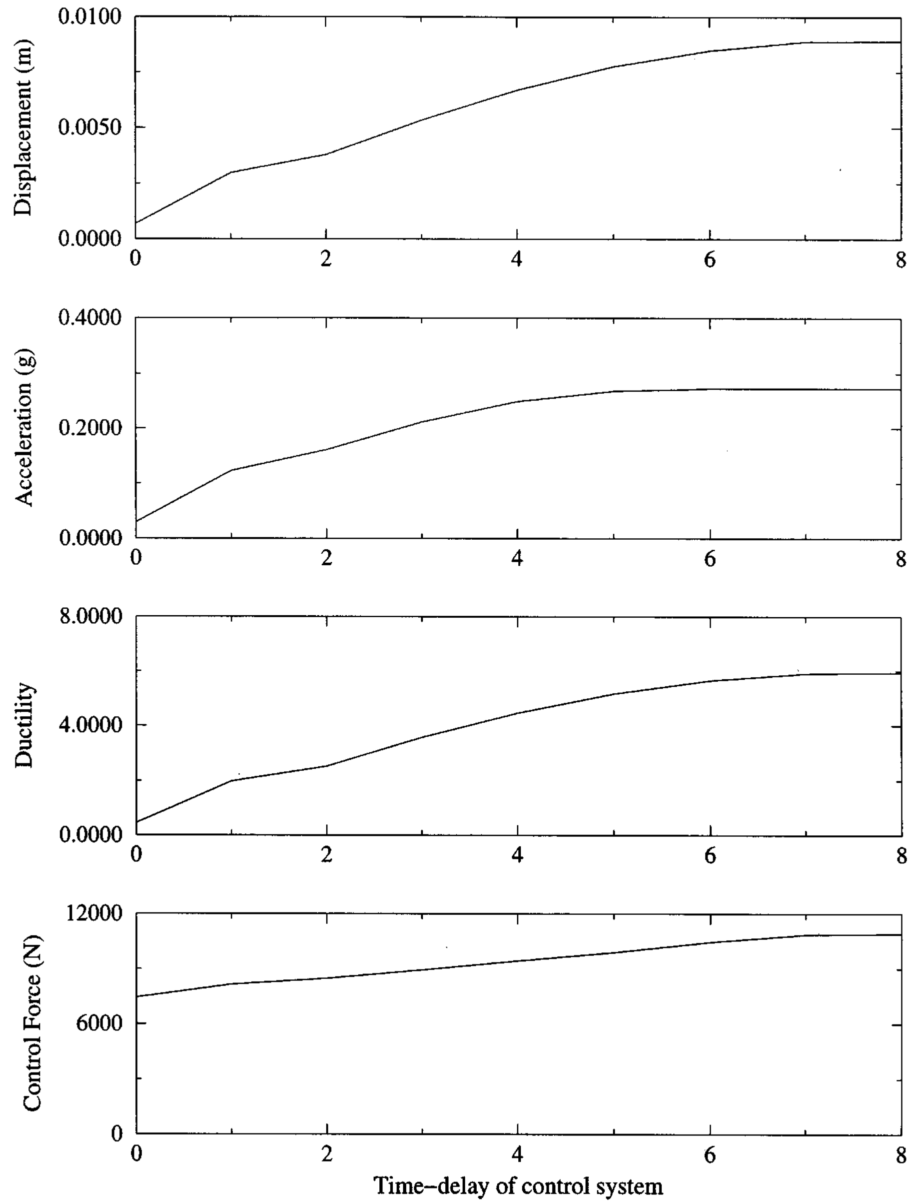
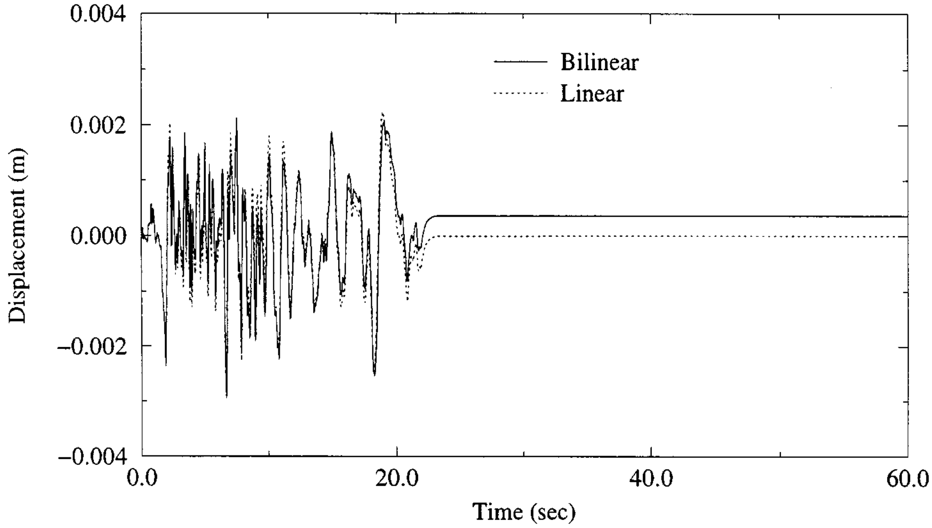


Figure 5. Maximum responses, ductility and control force versus time delay

When time delays are included, the effectiveness of the proposed control approach diminishes and there is a substantial worsening of the structural performances, as shown in Tables II and III and in Figure 5. Even a one-step time delay in either the measurements or the control action (Figure 6) is sufficient to push the structure beyond the elastic threshold, generating a substantial increment (about four times) in the values of the maximum structural responses. The maximum



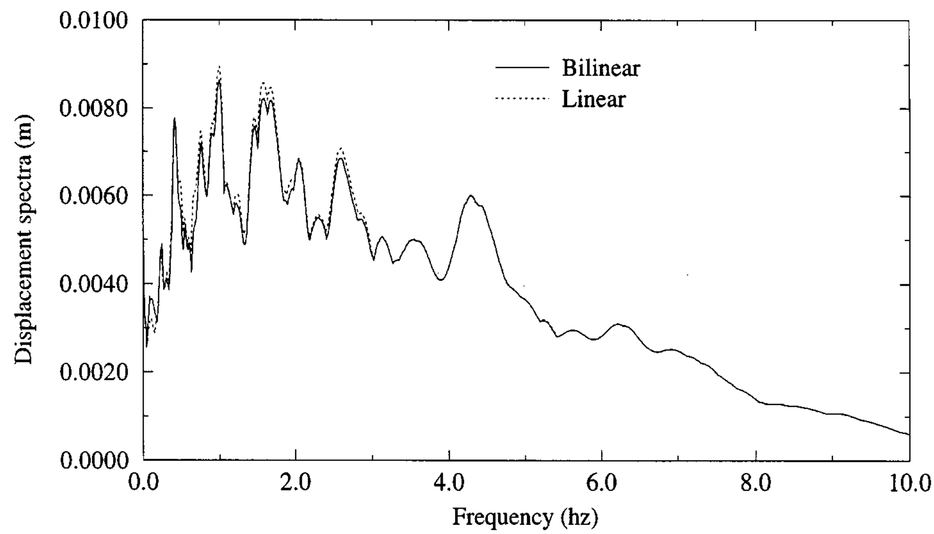


Figure 6. Structural response and spectra when time delay $= 1$

structural displacement increases from 0·00068 to 0·00295 m (Figure 6), while the maximum acceleration rises from 0·0295g to 0·124g. It is interesting to notice that the structure, even with the CONIOPC control system, presents now inelastic deformations, with a ductility almost equal to 2. As a consequence, the maximum control force shows a 4·5 per cent reduction with respect to the case of no time delay, confirming the fact that, when the structure enters into the non-linear range, energy is dissipated and the required control force is reduced. This can also be seen by confronting the first columns of Tables II and III. By comparing the case of linear and non-linear

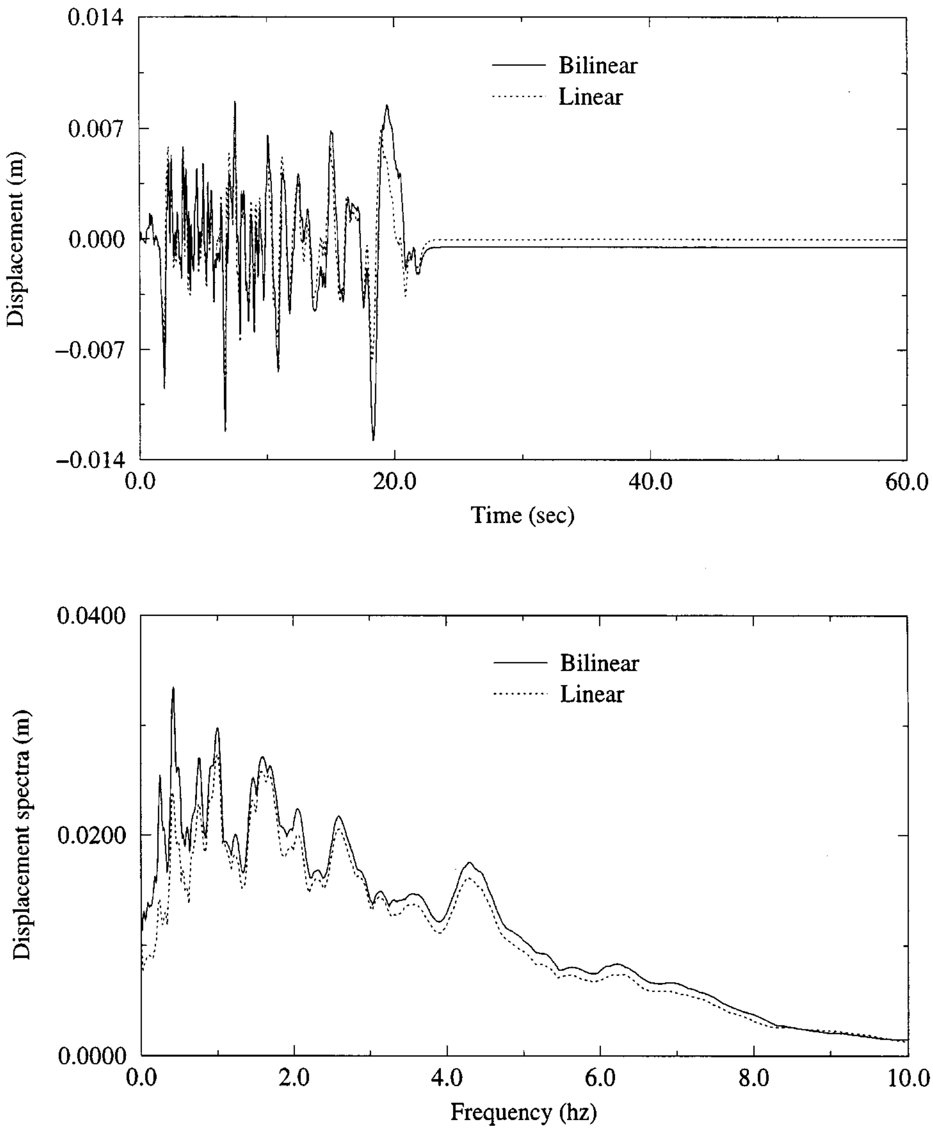


Figure 7. Structural response and spectra when time delays = 7

behaviour, the introduction of a 0·01 s time delay produces almost identical maximum structural performances (Figure 6). However, the energy dissipation effect in the non-linear case induces a lower maximum control force. Increasing the number of time delays, the effectiveness of the control system is drastically reduced, as shown in Tables II and III and Figures 5–8, for both linear and non-linear cases.

From a structural design point of view, it is very important to emphasize the effect of active control systems on structural ductility. The ductility capacity of a structure is directly related to

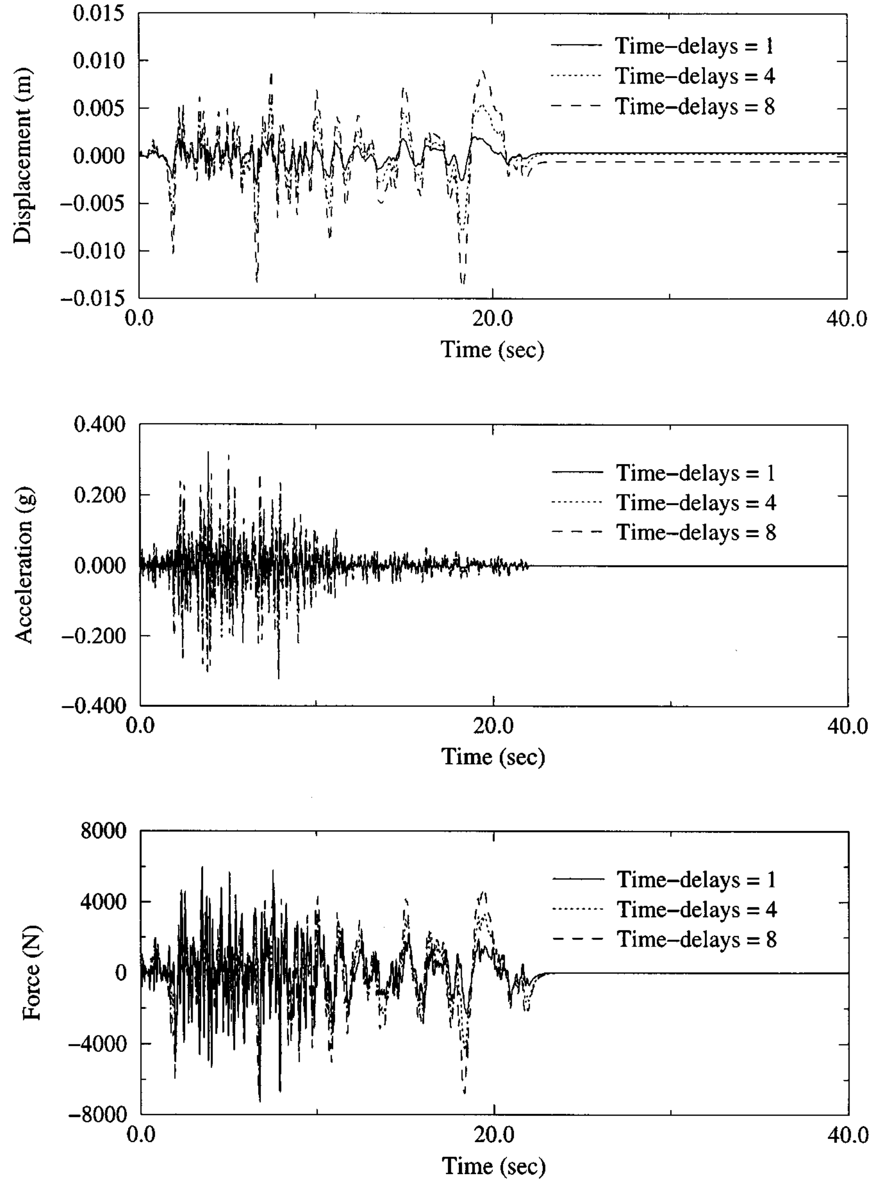


Figure 8. Structural responses and control force for bilinear structural model

the design and construction procedures and strongly affects the final cost of the structure. From Table I, it can be clearly seen that both the IOC solution and the CONIOPC solution dramatically reduce the ductility requirement, from a value of 15·73 (uncontrolled) to 1·148 (IOC). The proposed CONIOPC approach is capable of keeping the structure in the linear range, preventing the structure from any considerable damage. When time delay effects are included, the structural

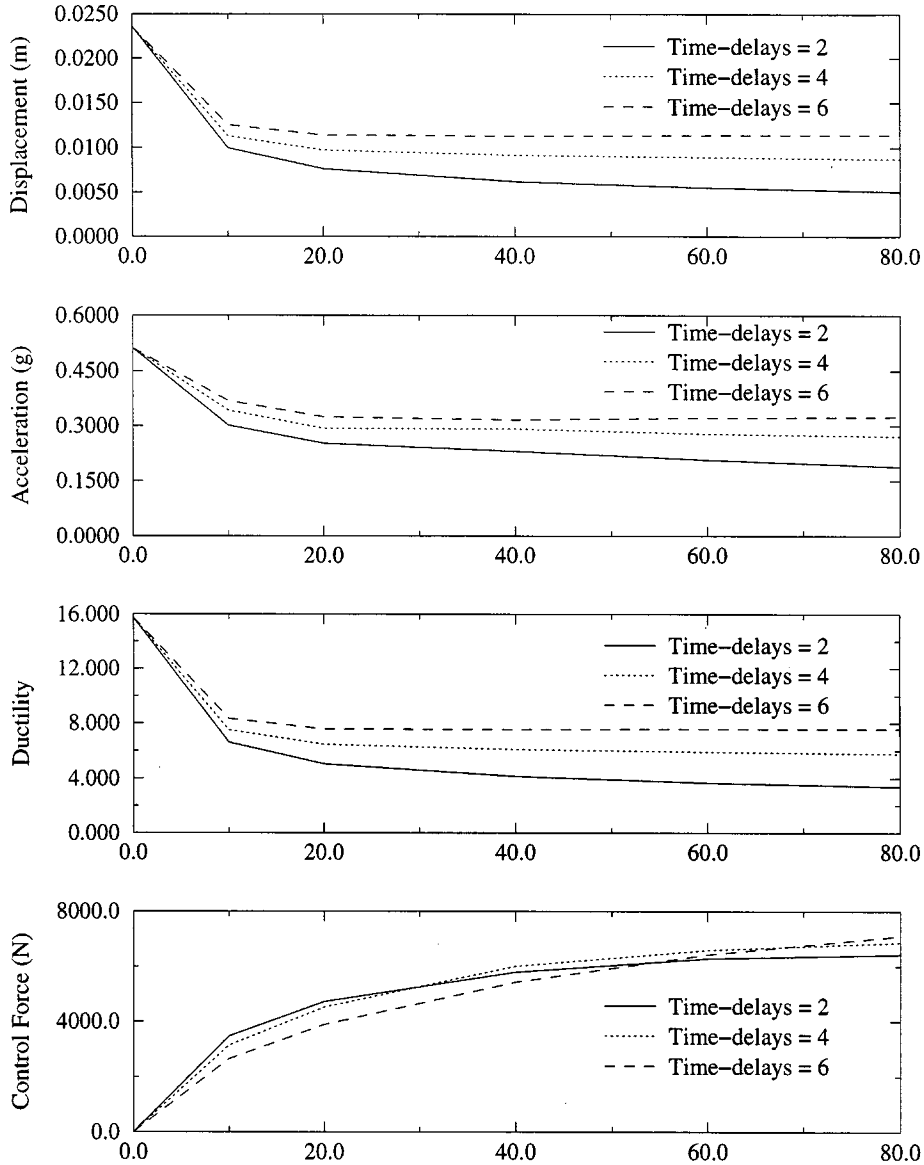


Figure 9. Maximum responses, ductility and control force versus $Q(2, 2)$ for bilinear structure with 2, 4, 6 time delays

response immediately shows non-linear characteristics, with a ductility requirement of 1·968 for a one-step time delay (Table III)). Together with the maximum structural response, the ductility requirements keep increasing as the number of time delay steps increases (Figure 5). This can be explained by the fact that, if more time is required for the measuring operation and for applying the control force, the predictive control action will prove less effective since more future state responses and ground excitation steps need to be predicted.

Table IV. Response quantities under different weighting matries (with 2 steps time delay)

$Q(2, 2)$	0	10	20	40	60	80	100
x (m)	0·02359	0·00991	0·00758	0·00620	0·00550	0·00507	0·00476
$\ddot{x}$ (g)	0·512	0·302	0·252	0·231	0·207	0·189	0·178
ρ	15·732	6·609	5·055	4·133	3·667	3·381	3·176
F_c (N)	0	3475	4724	5799	6277	6426	6567
Sp_x (m)	0·0698	0·0321	0·0238	0·0173	0·0146	0·0130	0·0120
$Sp_{\ddot{x}}$ (g)	1·69	1·65	1·42	1·15	0·98	0·87	0·80

Table V. Response quantities under different weighting matrices (with 6 steps time delay)

$Q(2, 2)$	0	10	20	40	60	80	100
x (m)	0·02359	0·01250	0·01140	0·01130	0·01137	0·01139	0·01137
$\ddot{x}$ (g)	0·512	0·368	0·324	0·317	0·322	0·324	0·325
ρ	15·732	8·345	7·568	7·537	7·584	7·594	7·582
F_c (N)	0	2649	3894	5433	6426	7103	7484
Sp_x (m)	0·0698	0·0421	0·0352	0·0298	0·0274	0·0271	0·0276
$Sp_{\ddot{x}}$ (g)	1·69	1·92	1·86	1·70	1·60	1·55	1·50

To show the importance of the time delay compensation, a dynamic analysis was conducted by simply applying the 'optimal' control force obtained without time delay compensation to the structure with a predetermined time lag. The results show that the control efficiency is reduced dramatically and even a two-step delayed control action induces instability in the structural ssystem. Therefore, time-delay compensation must be considered in advance in order to avoid instability problems from the control action.

4.4. Effects of relative magnitude of the weighting matrices

For optimal control methodologies, variability of the results depends on relative values of the weighting matrices Q and R and on the values of the matrices E and F. In addition, the choice of the matrix Q and F will affect the system stability, whose requirements impose Q to be positive semidefinite and both matrices to satisfy the Lyapunov stability condition. A proper choice of Q for an SDOF system is

$$Q = \begin{bmatrix} C_1 k & 0 \\ 0 & C_2 m \end{bmatrix} \tag{34}$$

which implies that the first part of the cost functional J_{k+1}^{p} represents the sum of the potential and kinetic energies of the structural system, with C_1 and C_2 as weighting constants. For the matrix F, equation (24) provides a suitable expression for the stability requirements. In order to estimate the relative importance of the four weighting matrices, we have selected the 1×1 R matrix as the reference one and looked at the variability of the results by changing one matrix at a time. For the active tendon system considered in this study, the value of $R(1, 1)$ has been chosen equal to $4k_c$, as proposed in Reference 1.

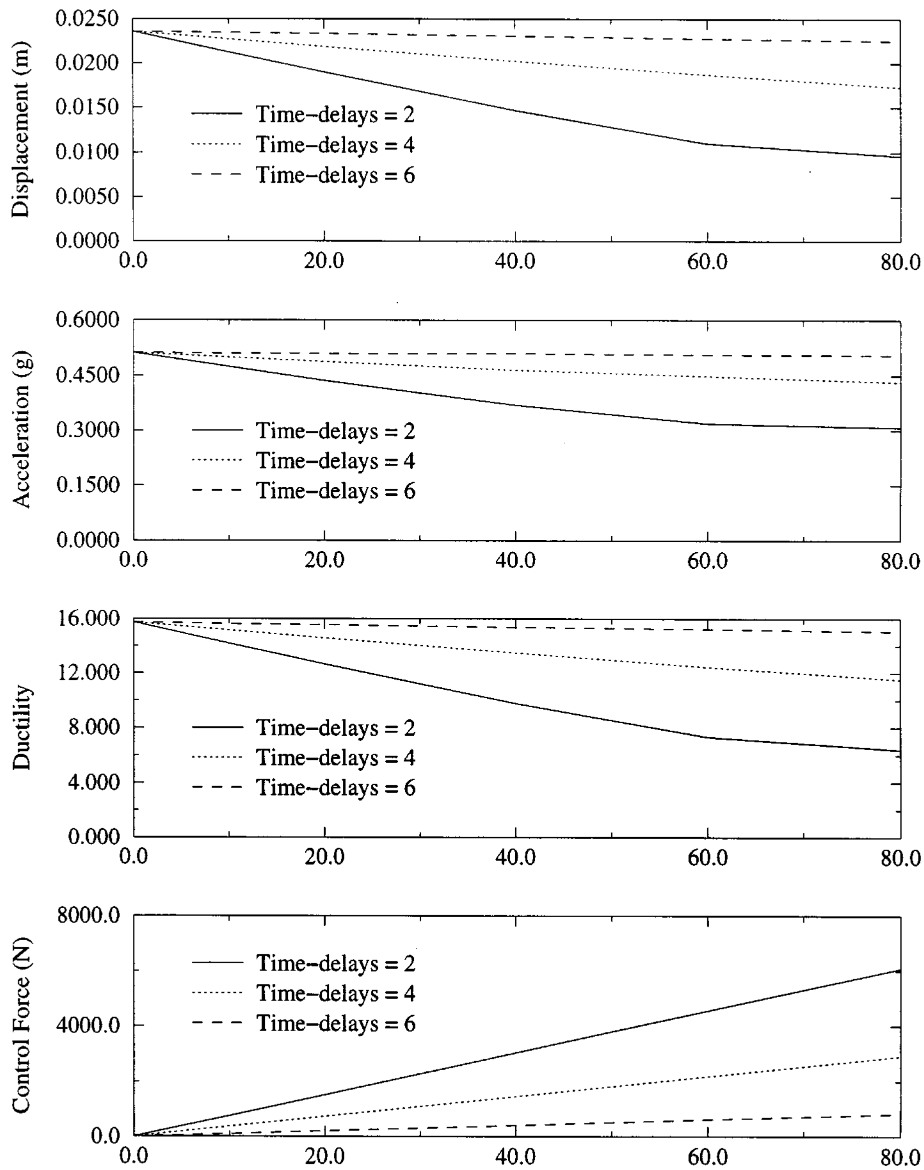


Figure 10. Maximum response, ductility and control force versus E for bilinear structure with 2, 4, 6 time delays

4.4.1. Relative magnitude of Q and R. Since $Q(1, 2) = 0$ an $Q(1, 1)$ does not have any contribution to the instantaneous optimal control force, the only term of the matrix Q which affects the final solution is the term associated with the structural velocity ($Q(2, 2)$). Since also the matrix R presents only one term, it is interesting to investigate the variability of the results by considering the relative value of $Q(2, 2)$ with respect to R. Figure 9 and Tables IV and V present the variability of the maximum displacements, accelerations, control force and ductility for the structural system

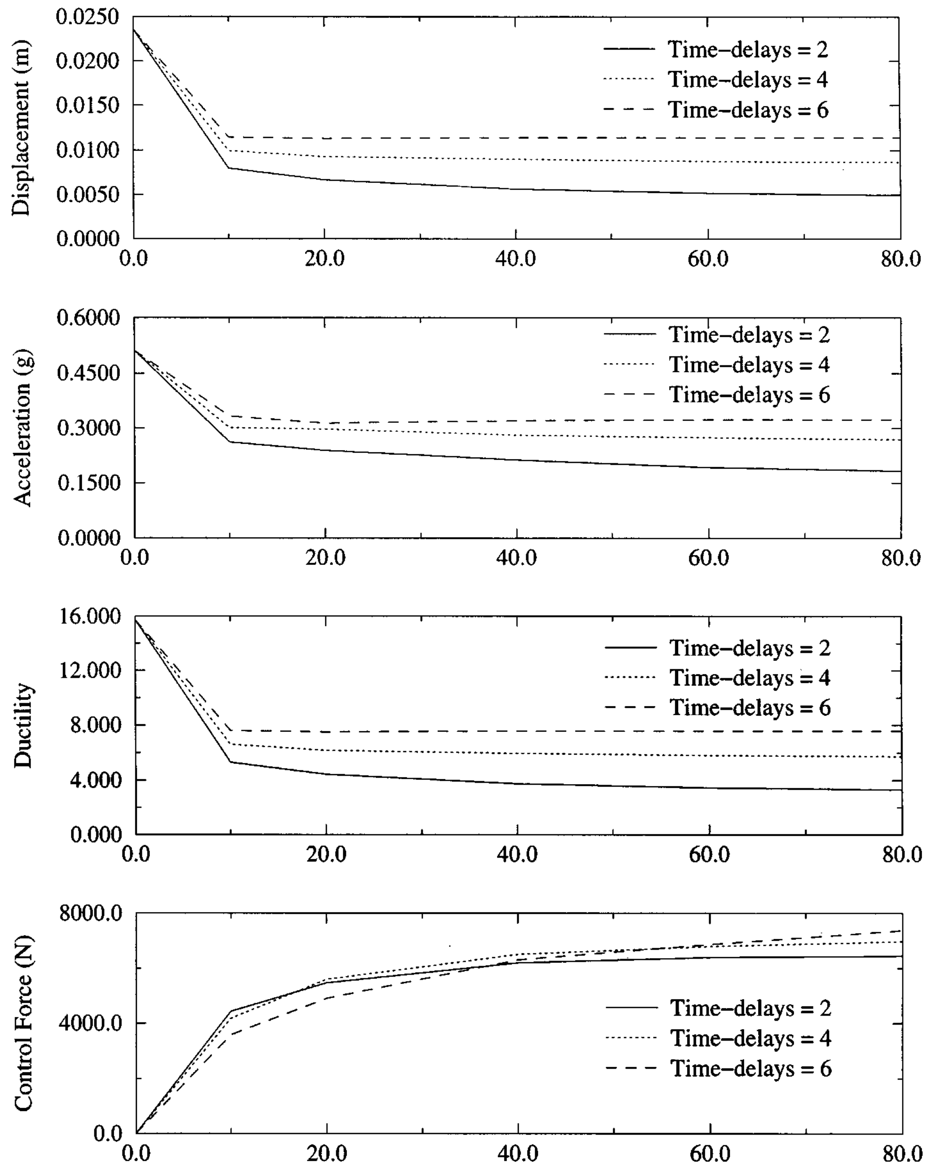


Figure 11. Maximum responses, ductility and control force versus F for bilinear structure with 2, 4, 6 time delays

as functions of the values of $Q(2, 2)$ and the number of time steps for the proposed CONIOPC algorithm. In order to emphasize the effects of the relative magnitude of Q and R, the matrices E and F have been assumed to be equal to null matrices. It is observed that, with a fixed time delay, as the value of $Q(2, 2)$ increases, all the structural response quantities decrease while the required control force increases. Both trends show an asymptotic behaviour whose value changes according to the number of time delay steps. As expected, when $Q(2, 2)$ is relatively small, the

effects of time-delay are not evident, while the magnitude of the control force is relatively small. Emphasizing the reduction in structural response, the magnitude of the control force increases and the effects associated with its asyncronous application are emphasized (Figure 9).

4.4.2. Relative magnitude of E and R. In order to analyse the effects of the relative magnitude of the matrix E, we assume that the matrices Q and F are null matrices. From equation (32), it can be seen that this control method will correspond to a pure open loop approach. From Figure 10, it is clear that, as the magnitude of E increases, all the structural responses and ductility decrease while the control action increases linearly. In addition, since the open-loop control term is directly related to the predicted ground acceleration, the time-delay effects become increasingly evident when more time delay steps are considered.

4.4.3. Relative magnitude of F and R. By imposing the matrix Q and E to be null matrices, the control algorithm reduces to a closed-loop component and allows us to analyse the effects of the relative magnitude between the matrices F and R. As in the previous cases, increasing the value of the matrix F generates a reduction in the structural response associated with an increase in the control action. The results are presented in Figure 11. By increasing the time lag, there is a clear worsening of the structural performances.

5. CONCLUSIONS

In this paper, a closed-open instantaneous optimal predictive control approach has been developed for the vibration control of non-linear hysteretic structures subjected to earthquake excitation. Such a control methodology includes (1) compensation of the effects due to time delays in both the response and excitation measurements and in the application of the control action and (2) the non-linear characteristics of the structural behaviour, accounting for the cumulative structural damage and for the energy dissipation. These two issues are of great importance in structural control of buildings in seismic areas since, in the current practice, (1) structures are assumed to perform non-linearly when subjected to strong earthquakes and (2) time delays are not avoidable in the response and excitation sensing and in the force actuating process. The proposed algorithm focuses on the instantaneous optimal control approach for the development of a control methodology where the non-linearities are brought into the analysis through a non-linear state vector and a non-linear open-loop term. An autoregressive (AR) model is used to predict the earthquake excitation to be considered in the prediction of the structural response. A performance index that is quadratic in the control force and in the predicted non-linear states, with two additional energy related terms, and that is subjected to a non-linear constraint equation, is minimized at every time step. The effectiveness of the proposed control algorithm is demonstrated by numerical simulations, showing that, through the use of controllers, the response of a non-linear structure to dynamic loadings can be reduced substantially and be kept within the linear range. It is further noticed that the proposed instantaneous optimal non-linear prediction control appears to be efficient when the time-delay compensation is included in the analysis. Although structural responses show a drastic increase with respect to the values corresponding to no time delay, the overall structural responses is still improved with respect to the uncontrolled case. In addition, when a nonlinear instantaneous optimal control approach is used, a strong reduction in ductility requirement is achieved, with great effects on the structural design and construction processes.

REFERENCES

1. T. T. Soong, *Active Structural Control*: *Theory and Practice*, Longman Scientific and Technical, U.K., 1990.
2. L. L. Chung, C. C. Lin and K. H. Lu, 'Time-delay control of structures', *Earthquake Eng. Struct. Dyn.* **24**, 687–701 (1995).
3. G. W. Housner, T. T. Soong and S. F. Masri, 'Second generation of active structural control in civil engineering', *Proc. 1st World Conf. on Structural Control*, Pasadena, CA, August, 1994, panel 1–18.
4. M. A. Soliman and W. H. Ray, 'Optimal feedback control for linear quadratic systems having time delays', *Int. J. Control* **15**, 609–615 (1972).
5. R. Mutharasan and R. Luus, 'Analysis of time-delay systems by series approximations', *Am. Inst Chem. Engrs. J.* **21**, 567–572 (1975).
6. L. G. Hammerstrom and S. K. Gros, 'Adaptation of optimal control theory to systems with time delays', *Int. J. Control* **32**, 320–357 (1980).
7. M. Abdel-Rohman, 'Time delay effects on actively damped systems' *J. Engng. Mech.* ASCE **113**, 1709–1719 (1987).
8. L. L. Chung, R. C. Lin, T. T. Soong and A. M. Reinhorn, 'Experimental study of active control for MDOF seismic structures', *J. Engng. Mech.* ASCE **115**, 1609–1627 (1989).
9. J. Rodellar, L. L. Chung, T. T. Soong and A. M. Reinhorn, 'Experimental digital control of structures', *J. Engng. Mech.* ASCE **115**, 1245–1261 (1989).
10. J. P. Pu and J. M. Kelly, 'Active control and seismic isolation', *J. Engng. Mech.* ASCE **117**, 2221–2236 (1991).
11. J. A. Inaudi and J. M. Kelly, 'A robust delay-compensation technique based on memory', *Earthquake Engng. Struct. Dyn.* **23**, 987–1001 (1994).
12. J. Roorda, 'Experiments in feedback control of structures', in H. H. Leipholz (ed.), *Structural Control*, North-Holland, Amsterdam, 1980, pp. 629–661.
13. L. L. Chung, A. M. Reinhorn and T. T. Soong, 'Experiments on active control of seismic structures', *J. Engng Mech.* ASCE **114**, 241–256 (1988).
14. S. McGreevy, T. T. Soong and A. M. Reinhorn, 'An experimental study of time-delay compensation in active structural control', *Proc. Int. Modal Analysis Conf.*, Orlando FL, 1989, pp. 733–739.
15. M. Abel-Mooty and J. Roorda, 'Time-delay compensation in active damping of structures', *J. Engng. Mech.* ASCE **117**, 2549–2570 (1991).
16. K. Huang, R. Betti and E. E. Ettouney, 'Instantaneous optimal control for seismic analysis of nonlinear structures', *J. Earthquake Engng.* **3**, 83–106 (1999).
17. M. Hoshiya and O. Maruyama, 'Identification of an autoregressive model for multivariate one-dimensional non-stationary gaussian random processes', *J. Proc. Japan Soc. Civil Engrs* **416**, 349–355 (1990).
18. T. Sato, K. Toki and M. Hashimoto, 'Active structural response control with self-learning mechanism, *Proc. Japan Soc. Civil Engrs* **471**, 115–124 (1993).
19. G. F. Panariello, R. Betti and R. W. Longman, 'Optimal structural control via training on an ensemble of earthquakes', *J. Engng. Mech.* ASCE **123**, 1170–1179 (1997).
20. W. H. Press, B. P. Flannery, S. A. Teukolsky and W. T. Vetterling, *Numerical Recipes*, *the Art of Scientific Computing*, Cambridge University Press, New York, 1989.
21. T. Sato and K. Toki, 'Active control of seismic response of structures', *J. Intell. Mater. Systems Struct.* **1**, 447–475 (1990).
22. K. Huang and R. Betti, 'Stable optimal control for seismically excited nonlinear and hysteretic structures', (1998) in preparation.
23. J. N. Yang, Z. Li and S. C. Liu, 'Stable controllers for instantaneous optimal control', *J. Engng. Mech.* ASCE **8**, 1612–1630 (1991).

